Toronto Prep School
Final Examination Review
Mathematics 10 (MPM2D)

Teacher: Mr. Tsimikalis
Date: Thursday, January 15, 2015
Time: 1:00 - 3:00 PM

Enclosed are sample questions ALL units .

The final exam will be composed of questions from each and every section/chapter.

The material on the Final Exam will only be from the material on the review sheet, you are not responsible for any material not on the sample questions.

Bring pencils, an eraser, a ruler, and a calculator (if you wish to use a graphing calculator you must bring your own and give it to the teacher at least 1/2 hour before the examination so it can be cleared).

A formula sheet will be supplied and it will include the following:

- length formula
- standard form
- SINE Law
- $ax^2 + bx + c = 0$
- midpoint formula
- quadratic formula
- COSINE Laws
- $y = a(x - h)^2 + k$
- slope formula
- vertex formula
- perimeter formula
- $a^2 + b^2 = c^2$
- y-intercept form
- SOHCAHTOA
- area formula
- $y = a(x - n)(x - m)$

You are responsible for:

Chapters 1 - 8

You must be able to:

Chapter 1:

- solve linear equations by:
  - graphing
  - substitution
  - elimination
- answer word problems using linear equations

Chapter 2:

- know when and how to use the appropriate formulas:

- determine length of a line segment given two end points
- know the difference between exact solution and approximate solution
- determine midpoint of a line segment given two endpoints
- determine endpoint of a line segment given one endpoint and midpoint
- write a linear equation in either y-intercept form or standard form given:
    - a point and a slope
    - two points on the line
- know the equation of a circle with origin at (0, 0)

Chapter 3:

- determine characteristics of triangles
    - median, right bisector, altitude
- verify characteristics of geometric shapes using algebraic techniques and analytical geometry
    - triangles, quadrilaterals

Chapter 4:

- function
    - determining what data set, description or graph is a function or not a function
- transformations to the standard parabola $y = x^2$
    - reflection
    - translation
    - dilation
- determining direction of opening, vertex, axis of symmetry, domain, range, min/max value, stretch/shrunk, narrower/wider and a sketch of the graph given graph $y = x^2$, $y = x^2 + k$, $y = ax^2$, $y = ax^2 + k$, $y = a(x - h)^2 + k$
- solving and graphing ax2 + bx + c = 0 by completeing the square
- sketching parabolas in the form y = a(x-m) (x-n)
- using first and second finite differences to determine if equation is linear, quadratic or neither
- negative and zero exponents

Chapter 5:

- add, subtract, multiple, divide and power terms and polynomials
- multiple binomials - FOIL
- factor polynomials
- factor $ax^2 + bx + c$, $a = 1$
- factor $ax^2 + bx + c$, $a \neq 1$
- use product/sum, breaking up middle and grouping

Chapter 6:

- solving quadratic equations by graphing
- solving quadratic equations by factoring
    - using zero-product rule
- solving word problems by factoring quadratic functions
- solving quadratic functions by using the quadratic formula
- solving word problems by using the quadratic formula
- graph quadratics using the x-intercepts
- writing $y = ax^2 + bx + c$ into $y = a(x - h)^2 + k$ form
- writing an equation in $y = ax^2 + bx + c$ form from a graph

Chapter 7:

- similar vs. congruent triangles
- using the concepts of similar triangles to determine angles, lengths and area
- determining what sine, cosine and tangent ratios are and what they mean
- using sine, cosine and tangent ratios to solve right angle triangles
    - SOHCAHTOA
- solving two right angle triangles

Chapter 8:

- using SINE LAW to solve triangles that are not right angle triangles
    - given 2 angles and 1 side
    - given 2 sides and an opposite angle
- using COSINE LAW to solve triangles that are not right angle triangles
    - given 2 sides and a contained angle
    - given 3 sides
- solving problems involving 2 right angles
- solve word problems using the above formulas

- You must know:
    - formulas for sine, cosine and tangent SOHCAHTOA
    - SINE LAW
    - COSINE LAW (both versions)
        - for angle
        - for side

1. Characteristics of linear systems (intersecting, parallel and distinct, and coincident lines)

   a) Coincident lines have this number of solutions _____

   b) Parallel and distinct lines have slopes that are _____

   c) Intersecting lines have this many solutions _____

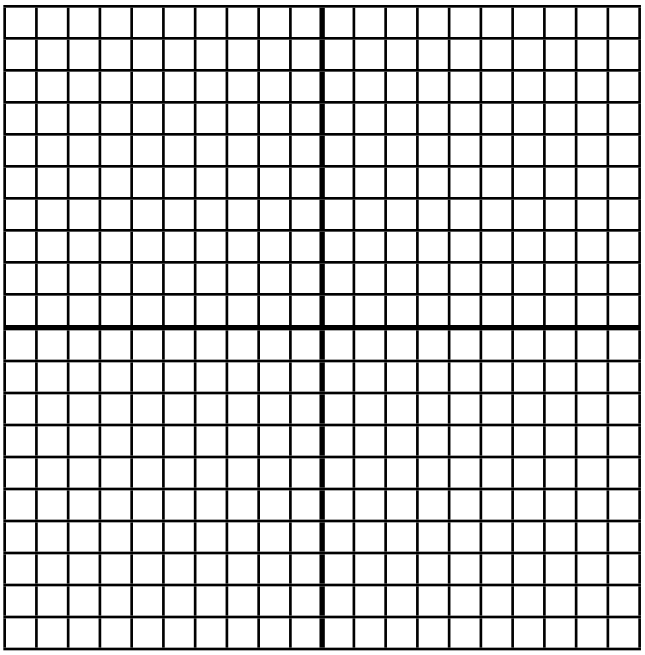
   d) Coincident lines have slopes that are _____

   e) Two lines with same slopes and different y-intercepts are _____

   f) Where is the solution for two intersecting lines found? _____

2. a) Solve the system by graphing. Your choice of graphing method. Check your solution.

$2x + 1 = y$
$-x - y + 4 = 0$



3. Solve the system by substitution. Check your solution.

$2x - 13 = y$
$x + 2y = -6$

4. Solve the system by elimination. Check your solution.

$2x + 3y = -5$
$-5x + 3y = -19$

5. a) Provide one way that an equivalent linear equation can be created

_____

_____

Do it for: $2x + 3y = -6$ _____

b) Provide one way you can create an equivalent linear system

_____

_____

Do it for: $2x + 3y = -4$ _____

$x + 2y = -3$ _____

6. Without graphing, determine whether these systems have one solution, no solution or infinitely many solutions. Explain your answer.

$y = \frac{-3x}{2} + 10$

$2y + 3x = 20$

7 Mr. T wins $16000 after the Vancouver Canucks destroy the Toronto Maple Laughs in a hockey game. He decides to invest part of his winnings in the stock market where he gets an 9% return and the remainder in bonds at 5% return. After one year Mr. T made $1240 in interest. How much money was invested in the stock market and how much was placed in bonds? Show your work.

8. Determine the midpoint of the line segment with endpoints (8, -4) and (12, -18).

9. For a line AB, one endpoint is A (-7, 4) and the midpoint is M (5, -2). Find the coordinates of endpoint B. You must use the midpoint formula and show your work.

10. Determine the length of the line segment joining points (-11, 6) and (-3, -5). Express the length as an exact solution (ES) and as an approximate solution (AS).

11. Determine the radius of the circle with equation $x^2 + y^2 = 121$

12. Write an equation for a line passing through points, A (3, 6) and B (5, 12).

13. Determine an equation for the line perpendicular to $4x + 2y - 7 = 0$ and passing through point (4, -10).

14. Find the shortest distance from P (-3, 5) to the line ½x + y - 6 = 0. Show your answer as an exact solution.

15. Mr. T has been kidnapped by the Anti-Algebra Alliance (AAA). He is being whisked away in a hot-air balloon. Mr. Oest takes out his trusty crossbow and aims it at the balloon in an attempt to knock it down and hence save Mr. T. Mr. Oest is standing at street level 10m west of the TPS sign in front of our building and the balloon with Mr. Oest in it is 21 m east of the sign and 29 metres above street level. How far must the crossbow bolt be fired if it is to strike the balloon? State it as an approximate solution. Include a diagram and coordinates in your diagram.

16. Having two items at your disposal; the length of a line formula and the slope formula, and your knowledge of parallel and perpendicular slopes, provide what you would do to prove (verify) that each of the following geometric shapes are indeed those shapes.

a) Parallelogram: _____
_____

b) Rectangle: _____
_____

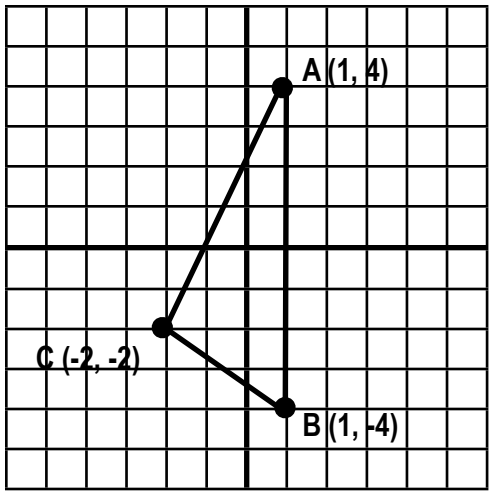
c) Isosceles triangle: _____
_____

d) Square: _____
_____

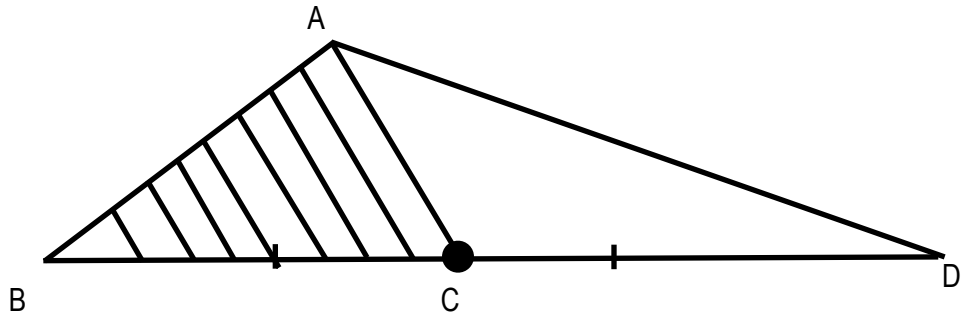
17. For the triangle drawn below complete the following:

a) What would you need to prove to verify that Δ ABC is a right angle triangle?

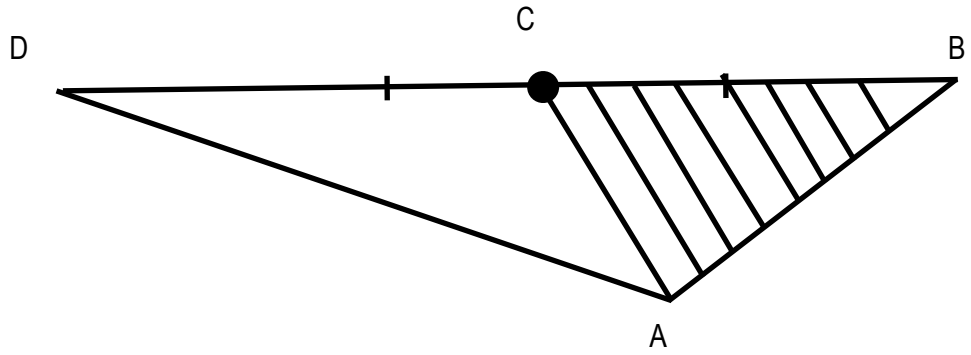
b) Verify that it is a right angle triangle



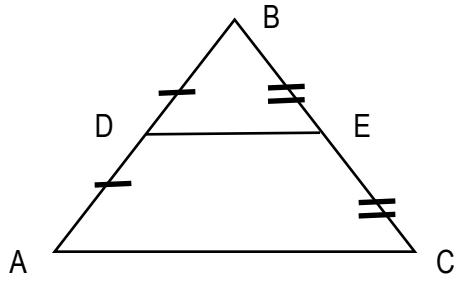
18. If Δ ABC has an area of 134 cm^2, the area of Δ ABD is _____ cm^2.



19. If Δ ABD has an area of 58 cm^2, the area of Δ is _____ cm^2.



20. Which one of the following statements is most correct given the diagram below? Circle the most correct one.

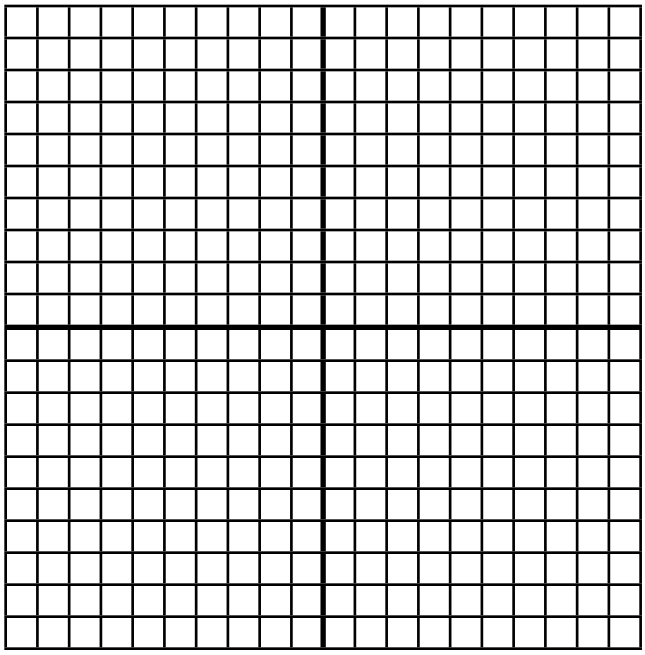


a) D and E are midpoints
b) DE is parallel to AC
c) the length of AC is exactly double the length of DE
d) all of the above are true
e) only two of a, b, and c are true

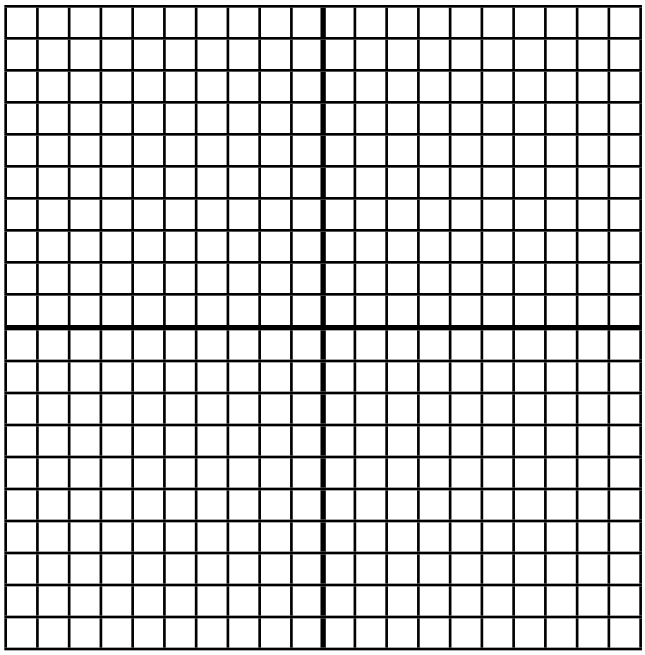
21. a) The Canadian War Museum recently bought a U2 Spy Plane used during the Cold War. They want to display it outside the Museum but are having difficulty mounting it on a stand. The U2 Spy Plane has a triangular shape and the museum wants to use a single mount coming from the ground up into the plane, balancing it so as to depict horizontal flight. direct the museum staff as to how they can find the balance point of the airplane. You may use a diagram to aid you in your explanation.

b) Differentiate between the centroid and the circumcentre of a triangle. How is each arrived at and what does it provide?

22. Verify that points A (-4, 2), B (5, -1) and C (3, -7) are the vertices of a scalene triangle which also happens to be a right angle triangle.

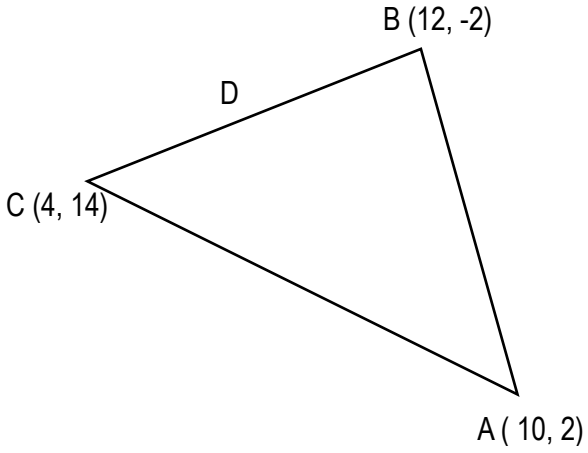


23. Verify that quadrilateral, A (1, 3), B (3, 2), C (2, -1) and D (-1, -3) is a trapezoid.



24. Verify that the quadrilateral formed by A (2, 4) B (3, 1) C (0, 0) and D (-1, 3) is a square. Be specific about all details of the verification.

25. a) Write an equation for the **median of AD**. Draw in the median include any pertinent information and label it.



b) State how the linear equation for the altitude of A and the right bisector of BC in the above triangle would be determined

26. Identify by proper name the type of transformation.

a) A mirror image of the figure (i.e. a flip) _____

b) A proportional enlargement or reduction, (i.e. a stretch by a factor of 7)
_____

c) A slide of the figure to the right or left or up or down (i.e. moved 3 left and 5 up)
_____

27. Using first and/or second differences determine whether each one of the following are linear equations, quadratic equations or neither.

a)

x	y
-3	1
-2	2
-1	4
0	8

_____

b)

x	y
-3	19
-2	15
-1	11
0	7

_____

c)

x	y
-3	9
-2	4
-1	1
0	0

_____

28. Write a quadratic equation in "useable" form, $y = a(x - h)^2 + k$ given the following information:

- vertex of (-2, 7)
- opens down
- is stretched by a factor of 3 _____

29. Write the equation for the parabola that has had the following transformation(s) done to the basic parabola of $y = x^2$. (3 marks each)

a) dilated (shrunk) by a factor of 1/3, and translated 4 left and 5 down

_____

b) reflected, dilated (stretched) by a factor of 4, translated 3 left and 5 up

_____

30. Provide the transformation(s) of each parabola compared to $y=x^2$. (2 marks each)

a) $y = x^2 - 6$ _____

b) $y = (x - 2)^2 + 7$ _____

c) $y = - (x + 3)^2$ _____

d) $y = -4x^2 - 3$ _____

31. For each function, state the direction of the opening, the coordinates of the vertex, the equation of the axis of symmetry the domain and range, the maximum or minimum value, whether it was stretched or shrunk, narrowed or widened and a sketch.

a) $y = -2(x - 3)^2 + 5$

opens _____

vertex _____

axis _____

domain _____

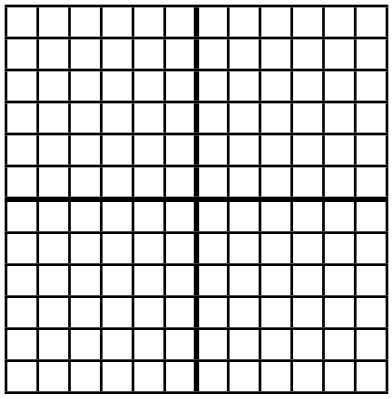
range _____

max/min _____

stretch/
shrunk_____

narrow/
wider _____

sketch



b) $y = 1/4x^2 - 2$

opens _____

vertex _____

axis _____

domain _____

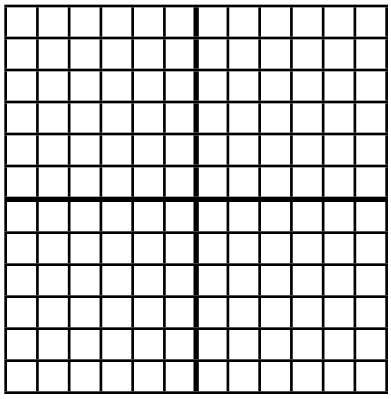
range _____

max/min _____

stretch/
shrunk_____

narrow/
wider _____

sketch



32. Given the equation and the fact that the parabola is 10 units in height, determine the x-intercepts, the vertex and 'a' and then write an equation in y = a(x - h)2 + k form.

y = (x - 8) (x + 2)

a) x-intercepts _____ and _____

b) vertex _____

c) 'a' _____

d) equation in $y = a(x - h)^2 + k$ _____

33. The following equation shows the flight path of a soccer ball kicked by Jonah to Julien, where 'h' is the height in metres and 'd' is the distance in metres from the kicker.

$h = -.03 (d - 25)^2 + 11$

a) What is the maximum height the ball attains? _____

b) How far down the field is the ball when it attains this height? _____

c) How far does the ball travel before it first hits the ground? _____

d) How high is the ball 7 metres from the kicker? _____

34. Evaluate each one of the following: Place final answer in fraction form.

a) 5^{-3}

b) 7^0

c) $(-4)^{-2}$

d) $\left(\frac{3}{4} \right)^{-3}$

35. Expand and simplify.

a) $5y(3y^2 - 2 + x)$

b) $2x^2(2x + 3y) - 5x(2x^2 + 4xy)$

36. Expand and simplify.

a) $(x - 4)(x + 3)$

b) $(4a + 3)(a - 5)$

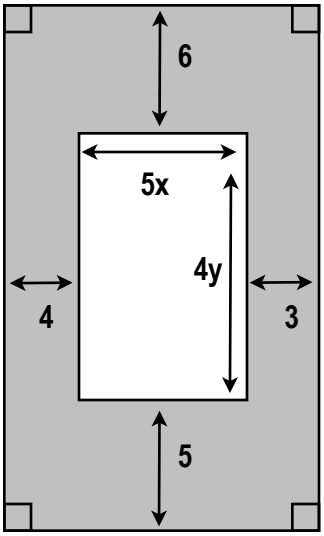
c) $(2x + 7)^2$

d) $(3x - 5)^2$

e) $3(5x - 2)(4x + 6)$

f) $2(3x + 4)^2$

37. i) Write a simplified expression for the area of shaded region.
ii) Calculate the shaded area if x = 3 and y = 4 (units are in metres)



Expression: _____

Area: _____

38. Factor.

a) $6x + 30$

b) $x^3 + 4x^2$

c) $5x + 4x^2 - 15 - 12x$

d) $6a^2b - 12a^2 + b^2 - 2b$

39. Factor, if possible. If not, possible, exhaust ALL the possibilities and then state ‘not possible’.

a) $x^2 + x - 42$

b) $x^2 + 8x + 12$

c) $x^2 - 5x + 36$

d) $x^2 - 169$

e) $8x^2 - 8x - 24$

f) $x^3 + 2x^2 - 48x$

g) $6x^2 - 5x - 4$

h) $15x^2 - 7x - 2$

i) $49a^2 - 81b^2$

40. Determine two values for c. Explain how you would do it.

a) $x^2 + 6x + c$ b) $x^2 - 2x + c$

For c and d, write in $y = a(x - h)^2 + k$ form by completing the square

c) $y = x^2 - 18x + 7$ d) $y = 3x^2 + 36x - 51$

41. A square seat cushion has a side length of $5x + 3$.

a) Write a simplified expression for the area of the top of the seat cushion.

b) If $x = 2$ what would the area of the top of the seat cushion be?

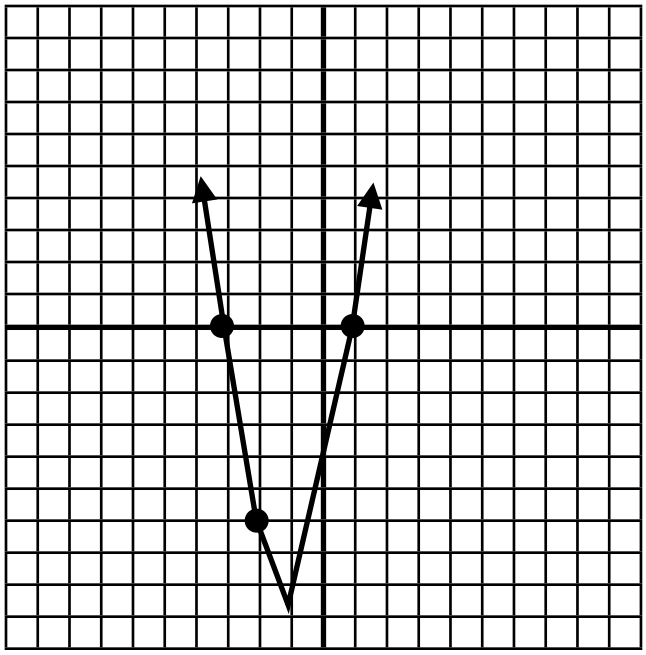
42. The top of a rectangular table has an area of is $x^2 + 11x - 12$.

a) Write an expression for the length and width of the table top

Length _____ Width _____

b) Determine the length and width if $x = 4$ Length _____ Width _____

43. Write an equation in $y = ax^2 + bx + c$ form for the parabola graphed below

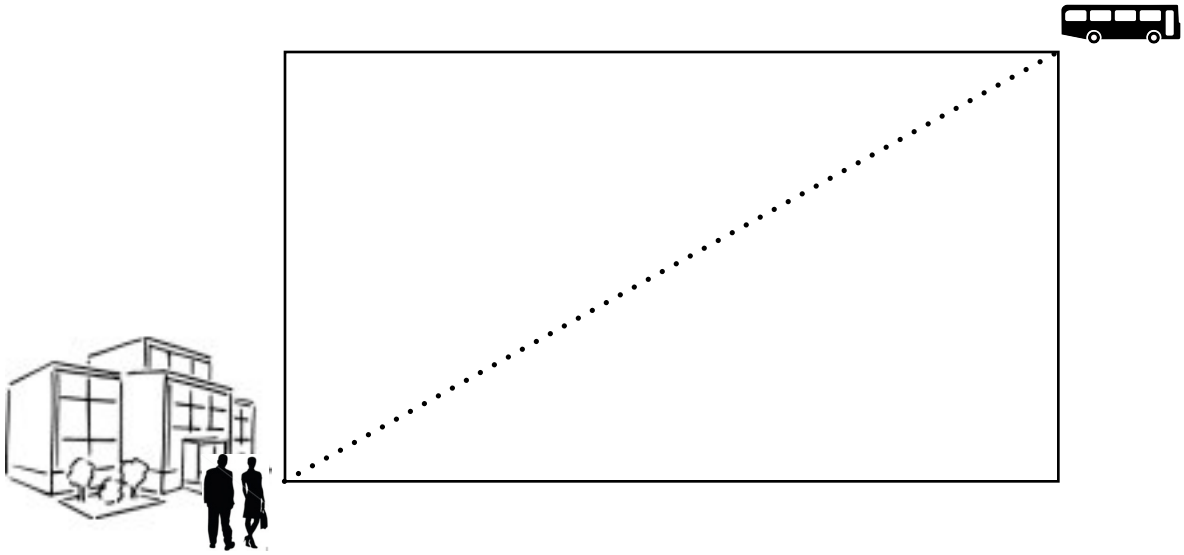


44. Find the vertex of the following quadratic by utilizing your knowledge of the quadratic formula:

$y = -8x^2 + 40x$

45. A garden has a floral arrangement 6 metres by 11 metres as its central area, with a grass border of uniform width around it. The total area of the garden is $93.75m^2$. Find the width of the grass border. Use the quadratic formula.

46. Mr. and Mrs. T decided to walk to the bus stop from the school. Mrs. T decides to trespass and takes the diagonal across a rectangular field walking a total distance of 48 metres to the bus stop. Mr. T "wimping out", decides to walk around the outside edge of the field. The length of the field is 13 m longer than its width. Use your method of choice to solve.



a) Provide the dimensions of the field _____

b) How far did Mr. T walk (to the nearest tenth of a metre)? _____

c) Who walked the least distance and by how much?

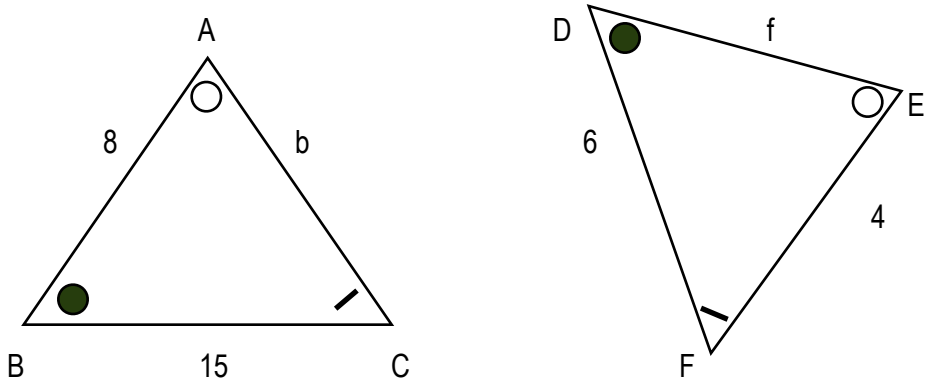
_____

47. A rectangle has a length of 9 metres and a width of 7 metres. If each dimension is increased by the same amount the area of the rectangle is 115 m^2. Find the dimensions of the new rectangle to the nearest tenth of a metre. Use your method of choice.

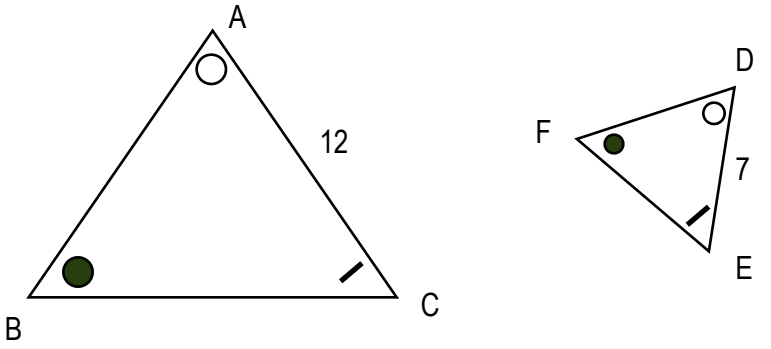
48. The hypotenuse of a right angle triangle is 4 metres shorter than twice the shortest arm. The longer arm is 4 metres longer than the shortest arm. Determine the lengths of each one of the arms. Use your method of choice. Include a diagram.

49. Isabel the photographer has a photograph 30 cm by 20 cm that she is going to crop. The same amount was cropped from its length and its width. The new area of the photo has been reduced by $199cm^2$. What are the new dimensions of the photograph to the nearest tenth of a centimetre. Use the method of your choice to solve.

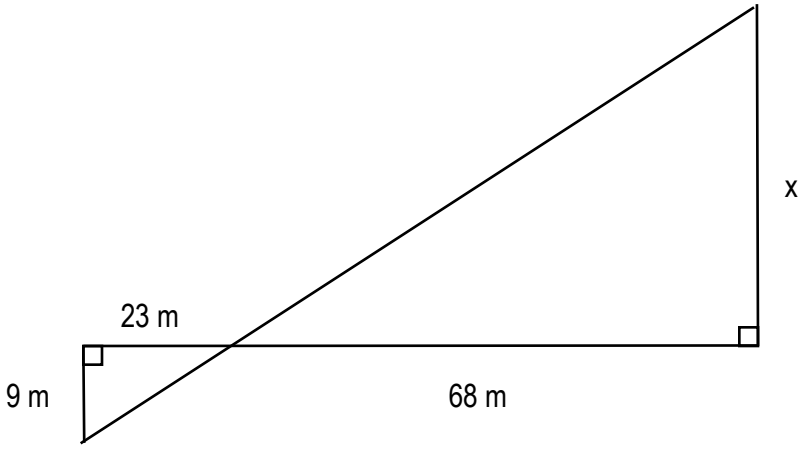
50. Δ ABC $\cong$ Δ DEF. Find the length of b and f.



51. Δ ABC ~ Δ DEF. The area of Δ ABC is 80cm^2, determine the area of Δ DEF.

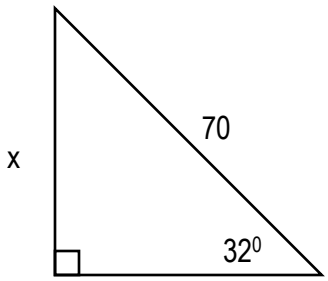


52. Determine the length of x.

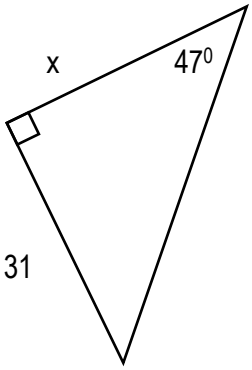


53. Determine the measure of x.

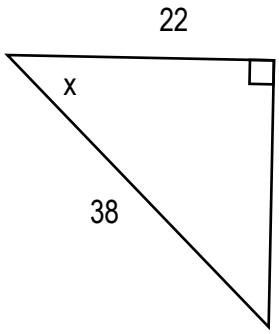
a)



b)

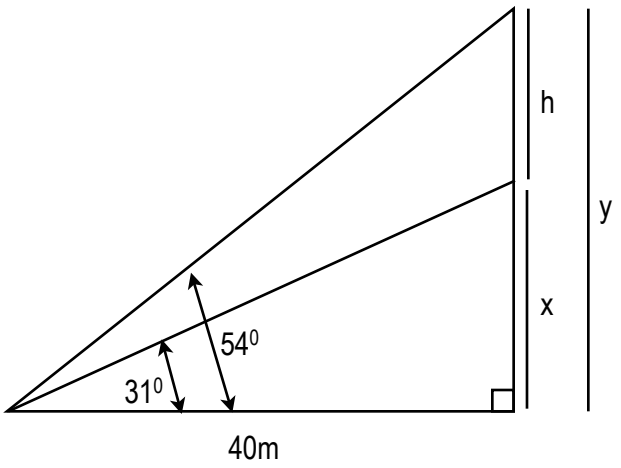


c)



54. The tallest tree in the TPS forest is a Sam Sequoia tree. From a point on the ground 42 m from the tree's base the angle of elevation to the top of the tree is 62^0. Find the height of the tree. Include a diagram as part of your answer.

55. Determine the height of h.



56. From the window of one building, Jack finds the angle of elevation of the top of a second building is 36^0 and the angle of depression to the bottom is 58^0. The buildings are 47m apart. Find to the nearest metre the following:

a) the height of the second building;
b) the height Jack was above the ground.

Include a diagram as part of your answer.

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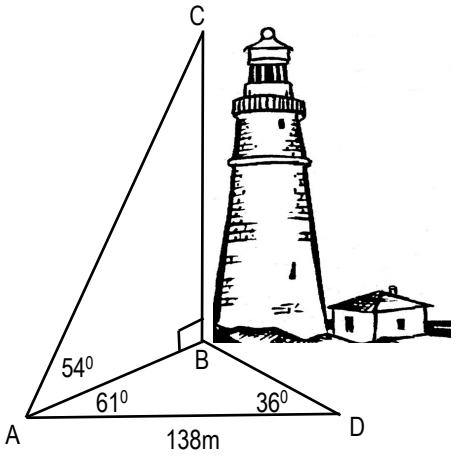
57. The Leaky Lighthouse found on Joel's Junction is a very popular tourist attraction. To find its height, ground measurements were taken as shown in the diagram. What is the height of the lighthouse to the nearest metre?



58. Two friends (Alex and Liam) left the TPS Stables on horseback. One travelled 16km per hour while the other travelled 20 km per hour. The angle separating the two was 47^0. How far apart were they after 4 1/2 hours? Include a diagram as part of your answer.